

Historic, Archive Document

Do not assume content reflects current scientific knowledge, policies, or practices.

Reserve
aGB980
.B73
1959

FOR ADMINISTRATIVE USE ONLY

Hydrograph

RESEARCH REPORT

No. 322

A NOTE ON A FUNCTIONAL DESCRIPTION OF RUNOFF HYDROGRAPHS

D. L. Brakensiek

Watershed Technology Research Branch
Soil and Water Conservation Research Division
Agricultural Research Service, USDA

December 4, 1959

A Note on a Functional Description of Runoff Hydrographs ^{1/}

D. L. Brakensiek ^{2/}

Hydrograph properties, such as volume of runoff, peak rate, time to peak and time-base are characterized by their respective numerical values. However, features such as the shape of the rising and falling side of the hydrograph must be characterized indirectly, e.g. by fitting a function to these segments the parameters of the function can then be used as numerical values. Plotting the falling side of the hydrograph on semi-logarithmic paper is one such technique. This attempts to represent the falling side by an exponential function and the parameter, slope of the best fitting straight line on semi-log paper, can then be used in further analysis. The many ramifications or variations of this technique are well known and need not be discussed here. The rising side of the hydrograph seems in general to be less amenable to analytical representation.

This note is not a rational development of a function. It simply presents one function which may have value for analytically representing a runoff hydrograph. Whether or not it has a place as a research technique will depend on whether the parameters can actually be related to measurable physical processes or systems. Of course, the first requirement for the representation of a hydrograph is that it have a fairly smooth shape and have a single peak. However, the function presented in this note may have some utility in describing multiple-peaked, but otherwise smooth, hydrographs.

^{1/} Contribution from the Watershed Technology Research Branch, Soil and Water Conservation Research Division, Agricultural Research Service, USDA, Beltsville, Maryland

^{2/} Hydraulic Engineer

DEVELOPMENT

The basic form of the function is

$$q(x) = (1/x)^n \text{EXP}(-b/\sqrt{x}) \quad (1)$$

where

$q(t)$ = rate at time t , in/hr

t = time, hrs.

n, b = parameters

EXP = the base of Napierian Logarithms

To standardize the function both sides are divided by $q(t=t_p)$, i.e., the peak rate,

$$q(x)/q(x_p) = 1/(x/x_p)^n \text{EXP}\{-b/\sqrt{x_p}\} [(1/\sqrt{x/x_p}) - 1] \quad (2)$$

Further, a simplification is possible by satisfying the boundary condition,

$$\frac{d[q(x)/q(x_p)]}{d(x/x_p)} = 0, \text{ when } (x/x_p) = 1$$

This insures that the function will reach a maximum at the same instant the runoff rate reaches its maximum. By the usual calculus technique for finding a maximum it is found that if

$$b = 2 n \sqrt{x_p}$$

the boundary condition would be satisfied.

Thus (2) now becomes

$$q(x)/q(x_p) = 1/(x/x_p)^n \text{EXP}\{-2n\} [(1/\sqrt{x/x_p}) - 1]$$

or

$$q(x)/q(x_p) = \text{EXP}\{-n \ln(x/x_p) - 2n[(1/\sqrt{x/x_p}) - 1]\} \quad (3)$$

A simple linear form is obtained by taking the natural logarithm of (3),

$$\ln [q(t)/q(t_p)] = -n \left\{ \ln (t/t_p) + 2 \left[(1/\sqrt{t/t_p}) - 1 \right] \right\} \quad (4)$$

As (4) has a multiplicative parameter the adequacy of the function can be tested graphical by plotting $[q(t)/q(t_p)]$ - values on semi-log paper with the bracketed right hand side as the new independent variable. To facilitate interpretation the bracketed right hand side can be considered as a transformed time scale,

$$T = \ln (t/t_p) + 2 \left[(1/\sqrt{t/t_p}) - 1 \right]$$

thus (4) becomes

$$\ln [q(t)/q(t_p)] = -nT$$

or
$$q(t)/q(t_p) = \text{EXP}(-nT)$$

Since all hydrographs are reduced to a common time scale, (t/t_p) , it would be possible to prepare a chart or tables so that T-values could be read directly. It also would be feasible to prepare semi-log paper with the T-scale superimposed on the arithmetic scale.

EXAMPLE

Table 1 presents the data for the example, the storm of April 24, 1957 on watershed W-1 at Riesel, Texas. The T-values were read from graphs prepared from the data of Table 2. Figure 1 shows the graph of $[q(t)/q(t_p)]$ versus T on semi-logarithmic paper and figure 2 presents the observed dimensionless hydrograph and the computed hydrograph. For this example the rising side was fairly accurately represented by a value of $n = 3.67$. The falling side showed a definite point of separation which indicated an initial n-value of 8.21 and a second value of 5.85. However figure 2 indicates that the entire falling side, for practical purposes, may be represented by $n = 8.21$.

Figure 3 illustrates, for the same storm as above, the usual practice of plotting the hydrograph recession on semi-logarithmic paper with the natural time scale (top scale) and also using the transformed time scale (bottom scale) presented in this note. The data appear to plot in a more regular manner when the T-scale is used and the break in slope is more distinct. Whether such a separation point has a physical meaning in terms of type of flow predominating remains for study.

CONCLUSIONS

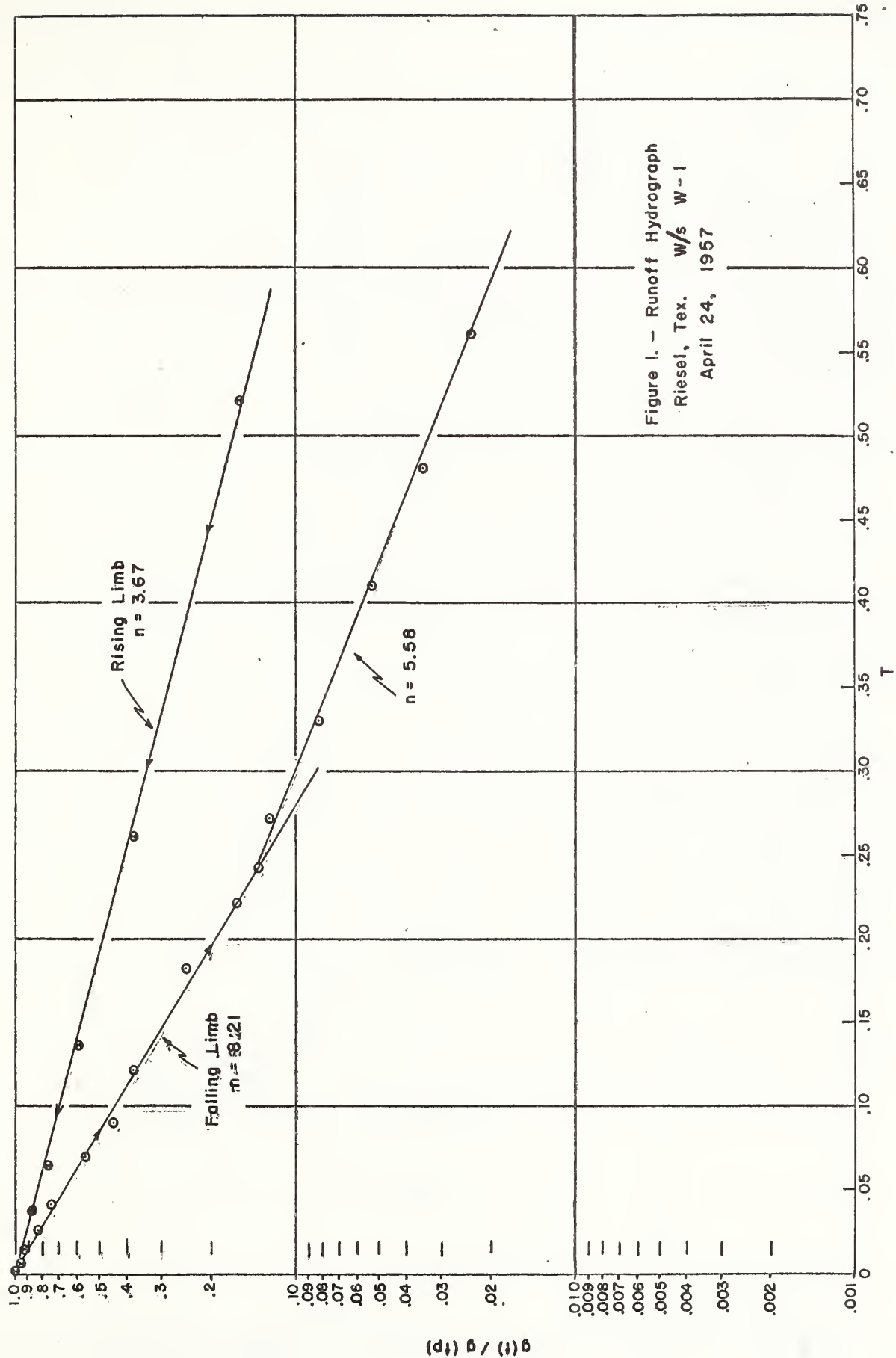
A function is presented as a possible tool for representing runoff hydrographs. Its value may lie in the ability to relate the parameter to measurable physical systems or processes. It also may aid in the study of hydrograph recession curves.

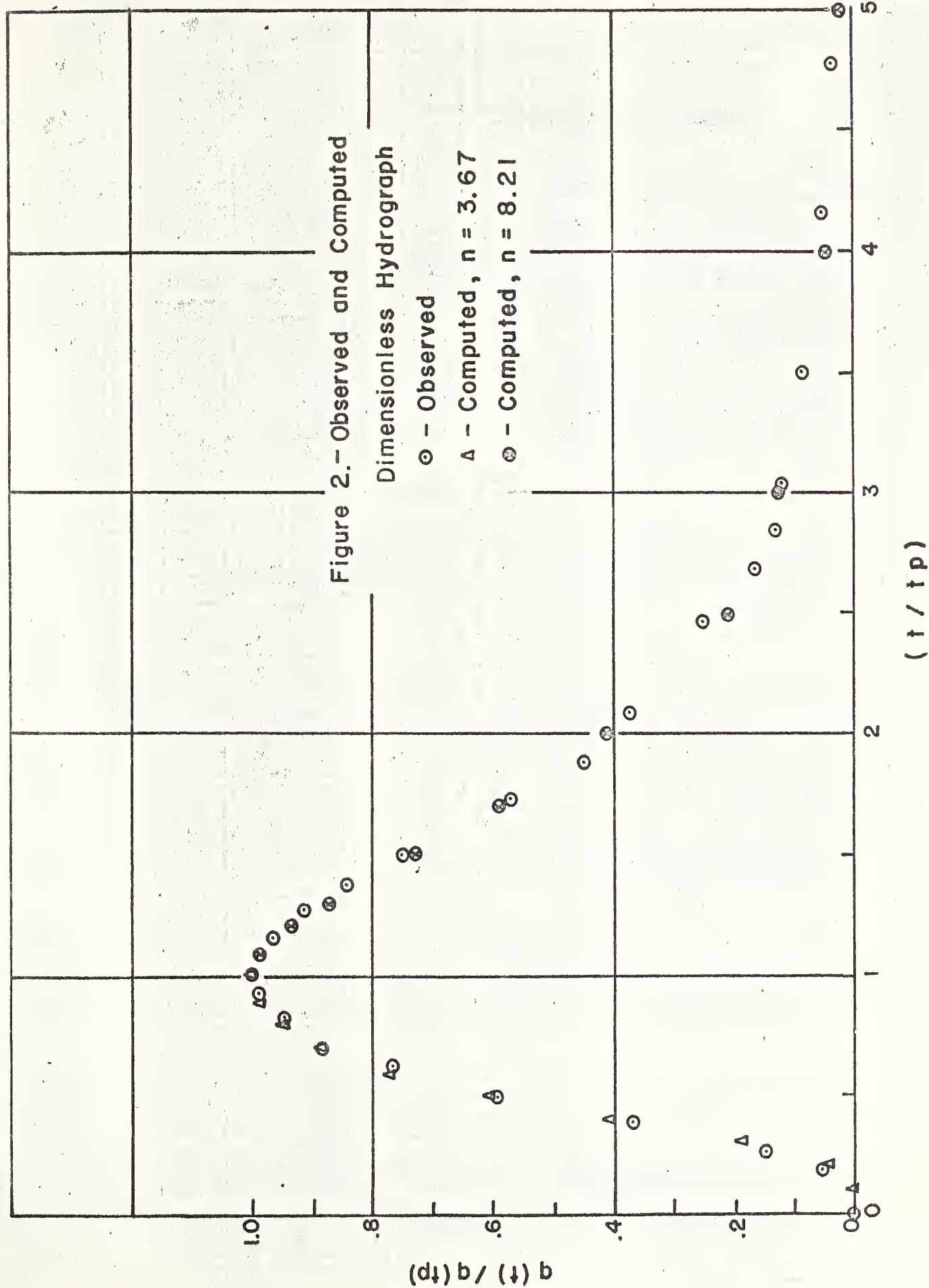
Table 1.--Storm of April 24, 1957 on
Watershed W-1, Riesel, Texas

<u>Clock Time</u>	<u>Time Lapse Min.</u>	<u>q(t) in/hr</u>	<u>(q/q_p)</u>	<u>(t/t_p)</u>	<u>T</u>
2:51 p	0	0.0056	0.0025	0	
52	1	.0144	.0065	.0385	$\bar{5}$
56	5	.130	.0591	.1923	.92
58	7	.342	.1555	.2692	.52
3:01	10	.830	.3773	.3846	.26
04	13	1.30	.5909	.5000	.135
07	16	1.68	.7636	.6154	.064
10	18	1.94	.8818	.6923	.036
12	21	2.08	.9455	.8077	.012
15	24	2.18	.9909	.9231	.002
17 (t _p)	26 (q _p)	2.20	1	1	0
19	28	2.17	.9864	1.0769	.0018
21	30	2.13	.9682	1.1538	.005
24	33	2.02	.9182	1.2692	.014
27	36	1.85	.8409	1.3846	.024
30	39	1.65	.7500	1.5000	.038
36	45	1.25	.5682	1.7308	.068
40	49	1.00	.4545	1.8846	.089
45	54	.824	.3745	2.0769	.12
55	64	.540	.2455	2.4615	.181
4:01	70	.353	.1605	2.6923	.22
05	74	.299	.1359	2.8462	.241
4:10	79	.268	.1218	3.0385	.27
22	91	.184	.0836	3.5000	.33
39	108	.115	.0523	4.1538	.41
55	124	.0756	.0344	4.7692	.48
5:15	144	.0509	.0231	5.5385	.56
6:50	239	.0144	.0065	9.1923	.88
8:00	309	.0072	.0033	11.8846	1.055
11:59	548	.0018	.0008	21.0769	1.483

Table 2.--Transformed time scale for a range of (t/t_p) -values.

t/t_p	T	t/t_p	T	t/t_p	T
0.01	13.39483	1.4	0.02678	5.5	0.55755
.02	8.22998	1.5	.03846	6.0	.60824
.03	6.04077	1.6	.05114	6.5	.65626
.04	4.78112	1.7	.06456	7.0	.70184
.05	3.94867	1.8	.07850	7.5	.74520
.06	3.35159	1.9	.09280	8.0	.78655
.07	2.90017	2.0	.10736	8.5	.82606
.08	2.54527	2.2	.13686	9.0	.86389
.09	2.25871	2.4	.16166	9.5	.90018
.1	2.02201	2.6	.19441	10.0	.93505
.2	.86266	2.8	.22485	11	1.00092
.3	.44750	3.0	.25331	12	1.06226
.4	.24601	3.2	.28119	13	1.11965
.5	.13529	3.4	.30843	14	1.17373
.6	.07117	3.6	.33502	15	1.22445
.7	.03382	3.8	.36098	16	1.27259
.8	.01294	4.0	.38629	17	1.31828
.9	.00282	4.2	.41098	18	1.36177
1.0	0	4.4	.43506	19	1.40327
1.1	.00224	4.6	.45857	20	1.44294
1.2	.00807	4.8	.48154		
1.3	.01649	5.0	.50387		





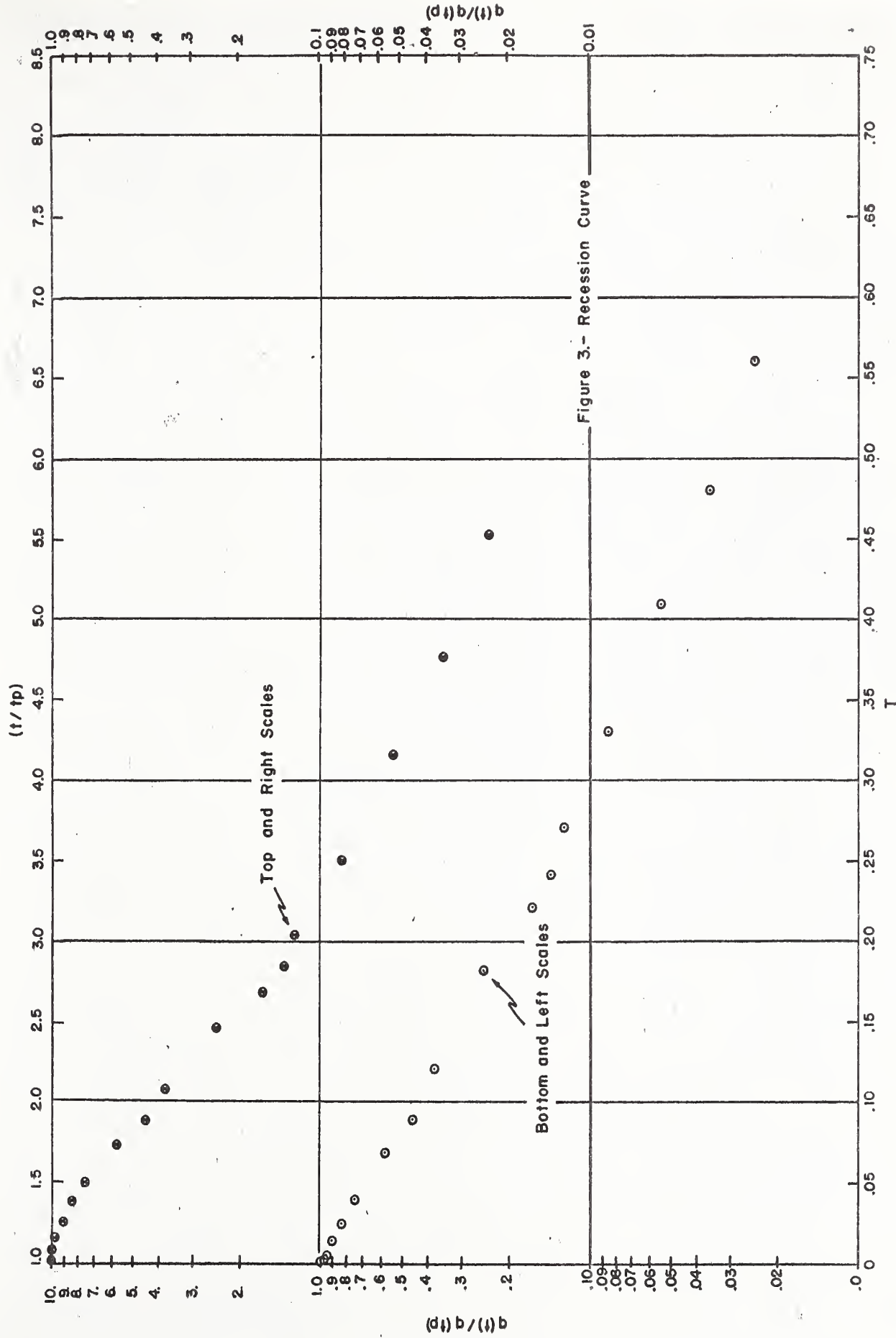


Figure 3.- Recession Curve

NATIONAL AGRICULTURAL LIBRARY



1023031797